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GROWTH OF A MAIN CRACK UNDER THE INFLUENCE OF GAS MOVING INSIDE IT

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UDC 622.011;622.023

The motion of a gas or liquid in a growing main crack is examined in connection with the problem of the hydraulic fracture of an oil-bearing bed [1, 2] and evaluation of the quantity of gaseous products escaping from the cavity formed by the underground explosion into the atmosphere by way of the crack [3]. The studies [1, 2] formulated and solved a problem on the quasisteady propagation of an axisymmetric crack in rock under the influence of an incompressible fluid pumped into the crack. An exact solution was obtained in [4] to the problem of the hydraulic fracture of an oil-bearing bed with a constant pressure along the crack. The Biot consolidation theory was used as the basis in [5] for an examination of the growth of a disk-shaped crack associated with hydraulic fracture of a porous bed saturated with fluid. A numerical solution to a similarity problem on the motion of a compressible gas in a plane crack was obtained in [6]. Here we examine the problem of the propagation of a main crack (plane and axisymmetric) under the influence of a gasmoving away from an underground cavity.

1. Formulation of the Problem. The motion of an isothermal gas in a main crack is described by the system of equations [6]

$$\frac{\partial}{\partial t}(\rho w) + \frac{1}{r^n}\frac{\partial}{\partial r}(r^n u \rho w) = 0, \qquad (1.1)$$
$$\frac{\partial}{\partial r} p + 12\mu w^{-2} u = 0, \quad p = c^2\rho,$$

where ρ is density; u is velocity; p is the gas pressure; c is the isothermal sonic velocity; μ is the gas velocity; n is a geometrical parameter (n = 0 for planar symmetry and n = 1 for axial symmetry); w is the opening of the crack; r is a coordinate; t is time.

Since the velocity of the crack under the influence of the gas moving inside it is much lower than the velocity of Rayleigh waves, then the opening of the crack is connected with the rock pressure and the gas pressure by the expression [7]

$$w(r, t) = \frac{4(1-\nu)}{\pi G} L(t) \int_{\xi}^{1} \int_{g(t)}^{\theta} \frac{[p(\eta, t) - p_\gamma]\, \eta^n d\eta\, \theta^{1-n} d\theta}{\sqrt{\theta^2 - \eta^2}\sqrt{\theta^2 - \xi^2}}. \qquad (1.2)$$

Here, $\xi = r/L(t)$; L(t) is the length of the crack at the moment of time t; $g(t) = r_0/L(t)$; r_0 is the radius of the underground cavity; p_γ is the rock pressure; ν is the Poisson's ratio; G is the shear modulus.

System (1.1)-(1.2) is closed by the condition of finiteness of the stresses on the contour of the crack [4], which determines the length (radius) of the crack:

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$$\int_{g(t)}^{1} \frac{[p(\eta, t) - p_\gamma]\, \eta^n d\eta}{\sqrt{1-\eta^2}} = \frac{K}{\sqrt{2L(t)}}. \tag{1.3}$$

It was shown in [8] that the effect of adhesive forces in the rocks can be ignored: $K/\sqrt{2L_0} \cong 0$, $L_0 = L(t = 0)$.

The initial and boundary conditions for system (1.1)-(1.3):

$$p(r, t = 0) = p_1, \ w(r, t = 0) = w_1(r), \tag{1.4}$$
$$p(r = r_0, t) = p_2(t), \ r_0 \leqslant r \leqslant L(t).$$

We introduce dimensionless variables and parameters by means of the formulas

$$R(t) = L(t) L_0^{-1}, \ L_0 = L(t = 0), \ \xi = rL^{-1}(t), \tag{1.5}$$
$$p'(\xi, t) = p(r, t) p_0^{-1}, \ p_0 = p_2(t = 0), \ t' = tt_0^{-1},$$

$$t_0 = L_0 u_0^{-1}, \ u_0 = \left(\frac{w_0}{L_0}\right)^2 \frac{p_0 L_0}{12\mu},$$

$$w_0 = L_0 \left[\frac{4(1-\nu)}{\pi} \frac{p_0}{G}\right] \frac{\pi}{2N} \ln(2\sqrt{e} N/\pi), \quad N = p_0/p_\gamma,$$
$$W(\xi, t) = w(r, t) w_0^{-1} R^{-1}(t), \quad U(\xi, t) = u(r, t) u_0^{-1} R^{-1}(t).$$

We will henceforth omit the primes, supposing that p = p' and t = t'. In dimensionless variables (1.5) system (1.1)-(1.3) and initial and boundary conditions (1.4), (1.5) have the form

$$\frac{\partial}{\partial t}(RWp) - \dot{R}\xi \frac{\partial}{\partial \xi}(Wp) + \frac{1}{\xi^n} \frac{\partial}{\partial \xi}(\xi^n RWp) = 0, \tag{1.6}$$
$$\frac{\partial}{\partial \xi} p + \frac{U}{W^2} = 0;$$

$$W(\xi, t) = \frac{2N}{\pi \ln(2\sqrt{e} N/\pi)} \int_{\xi}^{1} \int_{g(t)}^{\theta} \frac{[p(\eta, t) - N^{-1}]\, \eta^n d\eta \theta^{1-n} d\theta}{\sqrt{\theta^2 - \eta^2} \sqrt{\theta^2 - \xi^2}}; \tag{1.7}$$

$$\int_{g(t)}^{1} \frac{[p(\eta, t) - N^{-1}]\, \eta^n d\eta}{\sqrt{1-\eta^2}} = 0;$$

$$p(\xi, t = 0) = p_1/p_0, \ W(\xi, t = 0) = w_1(\xi L_0)/w_0, \tag{1.8}$$
$$p(\xi = g(t), t) = p_2(t)/p_0, \ g(t) \leqslant \xi \leqslant 1.$$

(1.9)

We represent the double integral (1.7) as

$$W(\xi, t) = \frac{2N}{\pi \ln(2\sqrt{e} N/\pi)} \int_{g(t)}^{1} d\theta Q_n(\xi, \theta) [p(\eta, t) - N^{-1}], \tag{1.10}$$

where

$$Q_n(\xi, \theta) = \theta^n \int_{\chi(\xi,\theta)}^{1} \frac{\eta^{1-n} d\eta}{\sqrt{\eta^2 - \theta^2} \sqrt{\eta^2 - \xi^2}}, \quad \chi(\xi, \theta) = \begin{cases} \xi, & \xi > \theta, \\ \theta, & \xi < \theta. \end{cases} \tag{1.11}$$

With n = 0, $Q_0(\xi, \theta)$ is expressed through elementary functions:

$$Q_0(\xi, \theta) = \ln \frac{\sqrt{1-\theta^2} + \sqrt{1-\xi^2}}{\sqrt{|\theta^2 - \xi^2|}}.$$

With n = 1, $Q_1(\xi, \theta)$ is an elliptical integral of the first type:

$$Q_1(\xi, \theta) = \theta\chi^{-1}(\xi, \theta) F\left[\arcsin\left(\frac{1-\xi^2}{1-\theta^2}\right)^{s/2}, \quad \left(\frac{\theta}{\xi}\right)^s\right],$$
$$s = \operatorname{sign}(\xi - \theta).$$

2. Method of Solution. We will use the decomposition method [9] to numerically solve problem (1.6)-(1.9). We introduce the following difference grid into the region being examined $t \in [0, \infty)$, $\xi \in [0, 1]$

$$\omega = \omega_\tau \times \omega_h = \{(\xi_k, t^m), \ \xi_{k+1} = \xi_k + h, \ \xi_{k+1/2} = (\xi_{k+1} + \xi_k)/2, \ t^{m+1} = t^m + \tau; \ k = 1, \ldots, M; \ m = 1, 2, \ldots\}.$$

System (1.6)-(1.8), decomposed at the differential level in terms of physical processes into three fractional time steps, has the form

at $t^m \le t < t^m + \tau/3$

$$\frac{1}{3}\frac{\partial}{\partial t} W = 0, \quad \frac{1}{3}\frac{\partial}{\partial t} R = 0, \tag{2.1}$$
$$\frac{1}{3}\frac{\partial}{\partial t} p - \frac{1}{W\xi^n}\frac{\partial}{\partial \xi}\left(\xi^n W^3 p \frac{\partial}{\partial \xi} p\right) = 0;$$

at $t^m + \tau/3 \leqslant t < t^m + 2\tau/3$

$$W = W[p], \quad R = R[p], \quad \frac{1}{3}\frac{\partial}{\partial t}(RWp) = 0; \tag{2.2}$$

at $t^m + 2\tau/3 \leqslant t < t^{m+1}$

$$\frac{1}{3}\frac{\partial}{\partial t} W = 0, \quad \frac{1}{3}\frac{\partial}{\partial t} R = 0, \quad \frac{1}{3}\frac{\partial}{\partial t} p - \frac{\dot{R}}{RW}\xi\frac{\partial}{\partial \xi}(Wp) = 0. \tag{2.3}$$

System (2.1) describe the motion of the gas in a stationary crack. The first and second relations in (2.2) coincide with Eqs. (1.7) and (1.8), respectively. We find from the third equation of (2.2) that $pRW = D(\xi)$ for $t \in [t^m + \tau/3, t^m + 2\tau/3)$ ($D(\xi)$ is a function of ξ). System (2.2) describes the propagation of a crack and the change in gas pressure in relation to the deformation of the crack. Equations (2.3) are connected with the change from a Eulerian coordinate system (r, t) to a moving system (ξ, t).

We introduce the grid functions p_k^m, W_k^m, R^m corresponding to the functions p, W, and R at the node (ξ_k, t^m) and $p_{k+1/2}^m$, $W_{k+1/2}^m$ at the node ($\xi_{k+1/2}$, t^m). We approximate the values of these functions at half-integral nodes by means of the formula $A_{k+1/2}^m = 0.5\left(A_{k+1/2}^m + A_k^m\right)$

Using the integro-interpolational method in [10], we obtain the following implicit difference scheme for three fractional steps from Eqs. (2.1)-(2.3)

1) $t^m \leqslant t < t^m + \tau/3$

$$W_k^{m+1/3} = W_k^m, \quad R^{m+1/3} = R^m, \tag{2.4}$$
$$\frac{p_k^{m+1/3} - p_k^m}{\tau} - \frac{1}{W_k^{m+1/3}\xi_k^n}\Lambda_k\{\xi^n W^3 p \Lambda_{k-1} p\}^{m+1/3} = 0;$$

2) $t^m + \tau/3 \leqslant t < t^m + 2\tau/3$

$$W_k^{m+2/3} = W_k\left[p^{m+2/3}\right], \quad R^{m+2/3} = R\left[p^{m+2/3}\right], \tag{2.5}$$
$$p_k^{m+2/3} W_k^{m+2/3} R^{m+2/3} = p_k^{m+1/3} W_k^{m+1/3} R^{m+1/3};$$

3) $t^m + 2\tau/3 \leqslant t < t^{m+1}$

$$W_k^{m+1} = W_k^{m+2/3}, \quad R^{m+1} = R^{m+2/3},$$
$$\frac{p_k^{m+1} - p_k^{m+2/3}}{\tau} - \frac{\xi_k}{R^{m+1} W_k^{m+1}}\left(\frac{R^{m+1} - R^m}{\tau}\right)\Lambda_k\{Wp\}^{m+1} = 0.$$

Here, $A_k^{m+p/3}$ ($p = 1, 2, 3$) is a grid function obtained as a result of satisfaction of the p-th fractional step; $\Lambda_k\{A\}^m = (A_{k+1}^m - A_k^m)/h$.

The integrals (1.7), (1.8) were calculated from the trapezoid formula. Here, the profile of the crack was determined from the expression

$$W_k^m = \frac{2N}{\pi \ln(2\sqrt{e}N/\pi)} \sum_{j=k}^{M-1} h\,(Q_n)_{k,j+1/2}\left[p_{j+1/2}^m - N^{-1}\right], \tag{2.6}$$

where $\ell \leq k \leq M$ and $\ell = [\overline{g(t)/h}]$ ($|\bar{z}|$ is the integral part of z). The index ℓ characterizes the coordinate of the boundary of the cavity ξ_ℓ on the grid ω from which the crack emerges at the moment of time t. The coordinate ξ_ℓ changes with the growth of the crack. Here, the crack remains stationary relative to the coordinate system (ξ, t).

The kernel Q_n does not depend on time, and the matrix $(Q_n)_{kj}$ (size $M \times M$, where M is the number of nodes in the grid) is calculated from Eq. (1.11) only once. The length of the crack in the m + 1-st time layer is determined from condition (1.8)

$$\int_{\alpha^{m+1}g^m}^{\alpha^{m+1}} \frac{\xi^n\left[p^{m+1} - N^{-1}\right]d\xi}{\sqrt{[\alpha^{m+1}]^2 - \xi^2}} = 0.$$

The parameter α^{m+1} is found by iteration from (2.6). We then determine the crack length $R^{m+1} = \alpha^{m+1}R^m$ in the m + 1-st time layer.

The mass balance was checked in the numerical solution

$$M_0(t) = 2^{n+3}\pi^{n-1}G^{-1}(1-\nu)L^{n+2}(t)\int_{g(t)}^{1}\int_{g(t)}^{1} \times$$
$$\times d\xi d\eta \xi^n \eta^n Q_n(\xi, \eta)\,p(\xi, t)\left[p(\eta, t) - N^{-1}\right]$$

($M_0(t)$ is the mass of the gas which has left the cavity).

The volume of the crack at the moment of time t is determined by the integral

$$v(t) = 2^{n+3}\pi^{n-1}G^{-1}(1-\nu)L^{n+2}(t)\int_{g(t)}^{1} d\xi\psi_n(\xi)\left[p(\xi, t) - N^{-1}\right],$$

where $\psi_n(\xi) = \int_{g(t)}^{1} d\eta Q_n(\xi, \eta)\,\eta^n \to \left(\frac{\pi}{2}\right)^n \sqrt{1-\xi^2}$

at $g(t) \to 0$.

The steady-state solutions of system (1.6)-(1.8) are similarity solutions of the problem of gas movement in a main crack (1.1)-(1.4) and can be obtained by the establishment method.

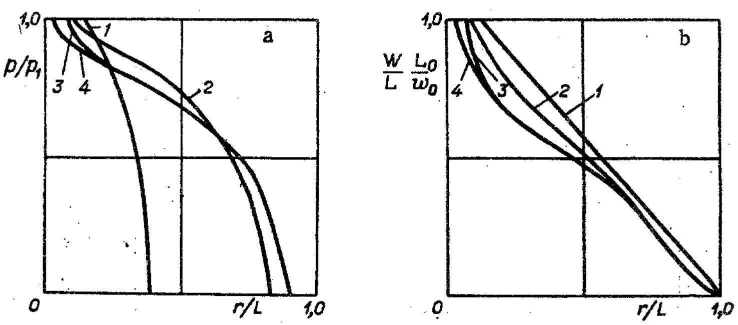

Fig. 1

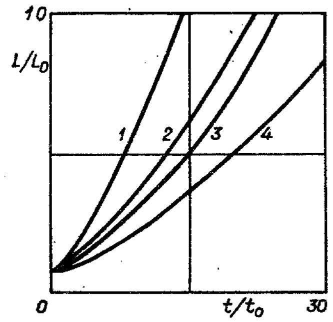

Fig. 2

3. Results and Discussion. The numerical solution yielded profiles of gas pressure in the cracks, crack length, and crack opening in relation to time.

Figure 1, a and b, shows the dependence of the gas pressure and crack profile on the coordinate at t = 0.1, 2.5, 10, and 15 (curves 1-4) for the axisymmetric case (n = 1) with the following parameters: $p_2(t) = p_0$, $p_1/p_0 = 0.01$, $\delta = r_0/L_0 = 0.1$, N = 4, $W_1(\xi) = 1 - \xi$ $(\delta \leq \xi \leq 1)$. It is evident that the profiles of gas pressure and crack opening at $t >> t^*$ (t^* is the time of the beginning of crack movement) depend only on ξ. This has to do with the fact that the gas–crack system forgets the initial conditions over time and asymptotically attains a similarity regime with the variable $\xi = r/L(t)$.

Figure 2 shows the dependence of the length of the propagating crack (n = 1) on time. Lines 2 and 4 correspond to N = 3 (δ = 0.5; 0.1); lines 1 and 3 correspond to N = 4 (δ = 0.5; 0.1).

The dynamics of the gas–crack system can be tentatively described in three stages: the first stage, at $t \in (0, t^*)$, is characterized by motion of the gas in a stationary deforming crack. The second stage ($t > t^*$) involves crack growth under the influence of the gas moving in the crack. The third stage ($t >> t^*$) corresponds to attainment of the similarity asymptote by the gas–crack system. Calculations showed that, as a function of time, crack length is approximated by the formula ($\tilde{t} >> t^*$):

$$L(t) \simeq L(\tilde{t}) \exp\{\beta(t - \tilde{t})\}. \tag{3.1}$$

Equation (3.1) follows from the similarity formulation of the problem ($\tilde{t} = 0$, $r_0 = 0$).

Table 1 shows parameters of similarity asymptotes for different values of N and n, where ξ_* is the coordinate of the gas front. With self-similar movement ($R(t) = \exp\{\beta t\}$, the variable is determined by the expression $\xi = r/\exp\{\beta t\}$. Here, the profiles of gas pressure and crack opening are shown in Fig. 3, a and b (n = 17, 10, 5, 2, and 1.5, curves 1-5) for the plane case and in Fig. 4, a and b (N = 17, 10, 3, and 1.5, curves 1-4) for the axisymmetric case.

It should be noted that the ratio of the gas pressure to the rock pressure is the determining parameter in the similarity problems (n = 0, n = 1). At $1 \leq N \lesssim 1.5$, the profiles of gas pressure and crack opening, as functions of the coordinates and time, are close to the results obtained in the solution of the hydraulic fracture problems in [1, 2, 4]. The compressibility of the gas begins to affect motion as N increases, which leads to a sharp drop in pressure near the coordinate origin.

Let us examine the propagation of a crack from a gas-filled cavity. The quasistatic solution of the problem of gas movement out of an underground cavity through a growing crack is applicable in the case when gas pressure in the cavity is greater than the lithostatic rock pressure at the end of the dynamic stage of the explosion. The excess pressure causes the gas to penetrate the main crack and create conditions for its deformation and growth.

As the boundary condition, we will use the condition of flow of gas from the cavity

$$\frac{\partial}{\partial t} M_C \Big|_{r=r_0} = - S\rho u \Big|_{r=r_0}. \tag{3.2}$$

Here, $M_C = 4/3\pi r_0^3 \rho$ is the mass of the gas in the cavity; r_0 is the radius of the cavity; $S = 2\pi r_0 w$ is the surface of gas flow from the underground cavity.

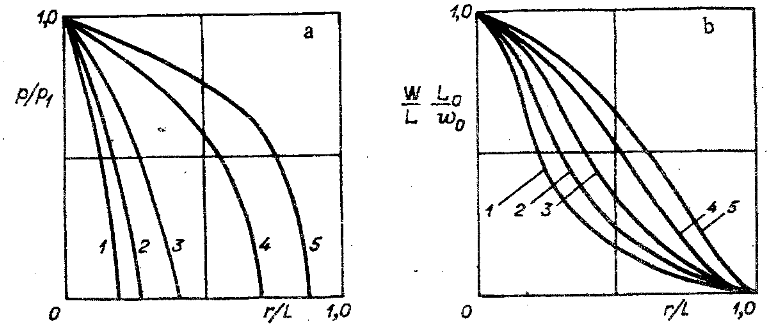

Fig. 3

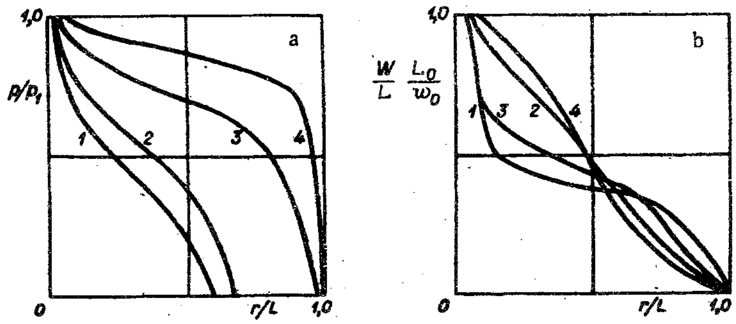

Fig. 4

TABLE 1

N	β	ξ_*	W(0)	β	ξ_*	W(0)
	n = 0			n = 1		
1,5	0,17	0,94	1,00	0,015	0,99	1
2,0	0,28	0,86	1,00	0,034	0,98	1
3,0	0,72	0,68	1,00	0,054	0,93	1
4,0	1,30	0,55	1,00	0,092	0,87	1
6,0	3,20	0,40	1,01	0,140	0,78	1
10,0	10,0	0,26	1,02	0,230	0,69	1
17,0	31,0	0,16	1,04	0,410	0,58	1

With dimensionless variables from (1.5), Eq. (3.2) takes the form

$$\frac{\partial}{\partial t} p = \varkappa R(t) W^3(t, \xi) p \frac{\partial}{\partial \xi} p, \xi = g(t),$$

where $\varkappa = \frac{6(1-\nu)}{\pi}\left(\frac{L_0}{r_0}\right)^2 \frac{p_\gamma}{G} \ln\left(2\sqrt{e}\, N/\pi\right)$ is the discharge coefficient.

Figures 5, a and b, shows the dependence of the gas pressure and crack-opening profile on the coordinate ξ at t = 9.5, 50, and 200 (curves 1-3) (N = 2, n = 1, p_1/p_0 = 0.01, $\delta = r_0/L_0$ = 0.1, κ = 0.1). In this case, the gas–crack system does not have a similarity asymptote.

Let us examine the effect of the parameters N, κ, and δ on the movement of gas from an underground cavity through a crack and let us determine their characteristic values. The study [11] examined the gas pressure at the end of the dynamic stage of expansion of the cavity:

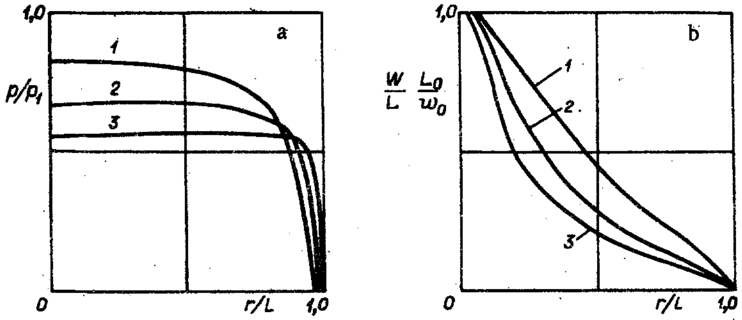

Fig. 5

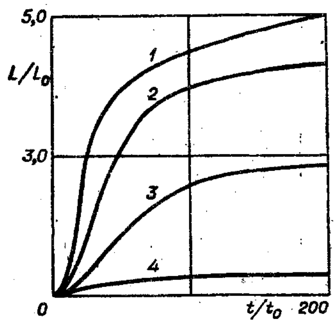

Fig. 6

$$N = p_0/p_\gamma = k,$$

where k is a coefficient dependent on the properties of the medium and ranging from 1.3 to 2.3 (for example, k = 2.1 for granite) [11].

Figure 6 shows the effect of the parameters κ and δ on the opening of the crack over time. Curves 1 and 3 correspond to $\kappa = 0.1$ ($\delta = 0.5$; 0.1), and curves 2 and 4 correspond to $\kappa = 0.2$ ($\delta = 0.5$; 0.1). Crack growth is determined by the ratio of p to p_γ.

The ratio of the radius of the cavity to the initial length of the main crack depends on the properties of the medium in which the cavity is formed [12]:

$$\delta \simeq \sqrt{\frac{\sigma_*}{2\sigma_0}}\left[\frac{E}{\sigma_*(q+1)}\right]^{\frac{1}{q+1}}, \quad q = \frac{2-\Lambda}{1+\Lambda},$$

where Λ is the dilatation rate (for an incompressible medium, $\Lambda = 0$, $q = 2$); σ_* is the strength in compression; σ_θ is the tensile strength; E is the Young's modulus.

At low values of δ, gas pressure in the cavity may be reduced so much that crack growth does not begin. Crack growth is significantly affected by the ratio of the maximum opening of a limit-equilibrium crack to the length of the crack ($W(\xi = g(t), t) \cong 1$):

$$\frac{w_0}{L_0} = 3\frac{\varkappa}{\delta^2} = \frac{2(1-\nu)}{\pi}\frac{p_\gamma}{G}\ln\left(2\sqrt{e}\,N/\pi\right).$$

At large values of w_0/L_0 (which is possible for rock with low shear moduli at high p_γ), outflow may reduce gas pressure in the cavity to values at which crack growth will not take place. Calculations showed that there is no crack growth at $\kappa \geq 0.2$, $\delta \leq 0.1$ but that crack growth does occur at $\kappa < 0.2$ or $\delta > 0.1$. Over time, the crack slows until it stops completely.

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STEADY-STATE HARMONIC ANTIPLANE VIBRATIONS OF A TWO-LAYER ELASTIC HALF-SPACE WITH A CYLINDRICAL CAVITY

S. O. Vorob'eva, A. A. Lyapin, and M. G. Seleznev

UDC 539.3

1. Formulation of a Boundary-Value Problem on Antiplane Steady Harmonic Vibrations. Let an elastic medium in a rectangular Cartesian coordinate system (x, y, z) occupy the region $x \geq -b$, $r = \sqrt{(x-h)^2 + y^2} \geq a$. A layer of thickness $b (-b \leq x \leq 0)$ with the parameters ρ, μ (ρ is density and μ is the shear modulus) is rigidly connected with the half-space $x \geq 0$. The half-space is characterized by the parameters ρ_1 and μ_1 and as a whole contains a horizontal cylindrical cavity of radius a with its center at the point (h, 0).

Distributed shearing forces are assigned on the boundary of the region, these forces undergoing steady harmonic oscillations over time with the frequency ω:

$$x = -b:\ \tau_{xz} = Z(y)\,e^{-i\omega t},\ r = a:\ \tau_{rz}^{(1)} = T(\varphi)\,e^{-i\omega t}. \tag{1.1}$$

Forces of rigid adhesion are assigned on the interface between the layer and half-space (x = 0), these forces determining the equality of the displacements (w(x, y)) and the shearing stresses τ_{xz}:

$$\begin{gathered} w(x, y)|_{x\to-0} = w^{(1)}(x, y)|_{x\to+0}, \\ \tau_{xz}(x, y)|_{x\to-0} = \tau_{xz}(x, y)|_{x\to+0}. \end{gathered} \tag{1.2}$$

Here and below, the superscript (1) denotes characteristics of the half-space. The motion of the medium is described by the dynamical equations of the theory of elasticity in displacements — the Lame equations [1]. We will seek to solve the formulated boundary-value problem in the class of integrable functions.

We designate the contact stresses on the interface as follows

$$x = 0:\ \tau_{xz}(0, y, t) = R(y)\,e^{-i\omega t} = \tau_{xz}^{(1)}(0, y, t). \tag{1.3}$$

In this case, we will use the method of Fourier transformation to solve the boundary-value problem for an elastic layer $-b \leq x \leq 0$ with boundary conditions (1.1), (1.3). Here, the

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